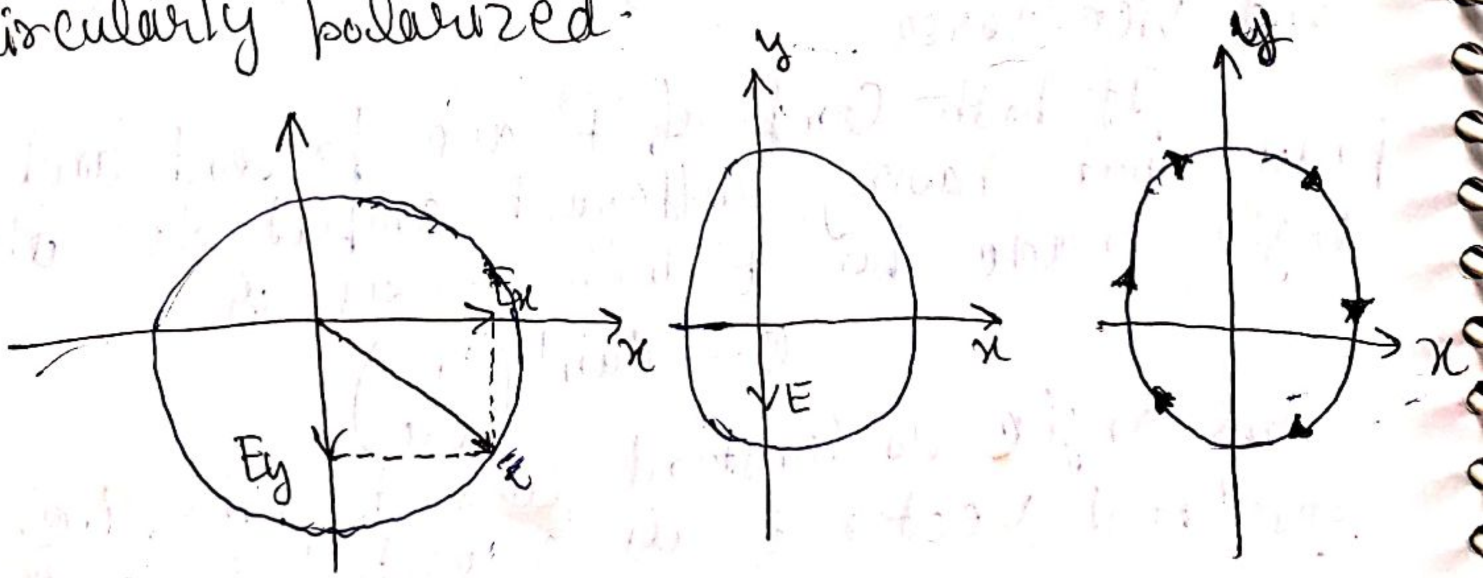
Circular polarization:— Let, $\vec{E}$ has two components E_x and E_y of equal amplitude but phase difference b/w them is $\pi/2$. Therefore, at any instant of time the magnitude of Resultant vector $\vec{E}$ is Constant but direction Changes with respect to time.

If E is projected on a plane perpendicular to the direction of propogation then locus of all points forms a circle with centre on z-axis. In one wave length span the resultant vector $\vec{E}$ Completes one cycle of Rotation such a wave is said to be circularly polarized.



$$\vec{E} = \vec{E_x} \sin(\omega t - Kz)\hat{x} + \vec{E_y} \cdot \sin(\omega t - Kz + \delta)\hat{y} \quad \text{—— (1)}$$

δ is phase shift b/w two components if $\vec{E}$ is observed at $z=0$,

$$\vec{E} = \vec{E_x} \sin\omega t\, \hat{x} + \vec{E_y} \sin(\omega t + \delta)\, \hat{y} \quad \text{—— (2)}$$

Now, we may arises ~~two~~ cases:-

Case (i) -

$\delta = \pi/2$

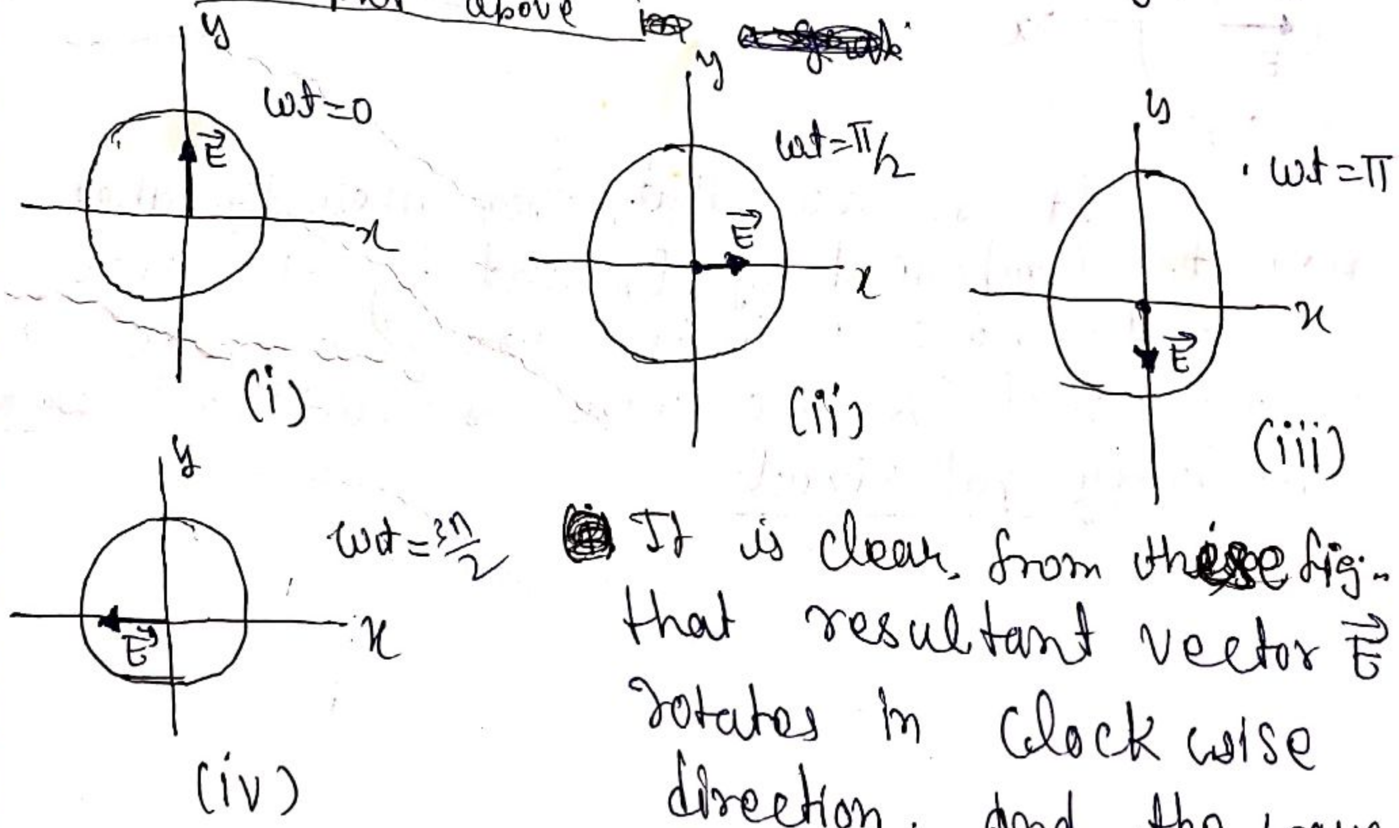
If $\omega t = 0$, $|\vec{E_x}| = 0$ & $|\vec{E_y}| = E_y$

if, $\omega t = \pi/2$ $|\vec{E_x}| = E_x$ & $|\vec{E_y}| = 0$

if, $\omega t = \pi$ $|\vec{E_x}| = 0$ & $|\vec{E_y}| = -E_y$

If, $\omega t = 3\pi/2$ $|\vec{E_x}| = -E_x$ & $|\vec{E_y}| = 0$

We plot above ~~in~~ ~~a graph~~



It is clear from these fig. that resultant vector $\vec{E}$ rotates in clock wise direction. And the wave is said to left circularly polarized.

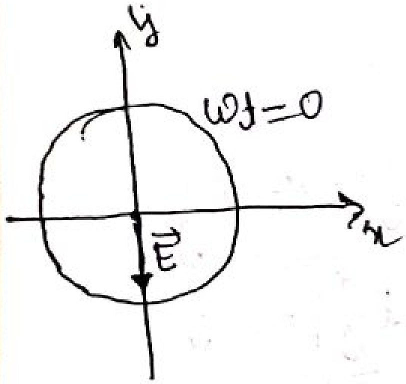
Case (iii) - $\delta = -\pi/2$

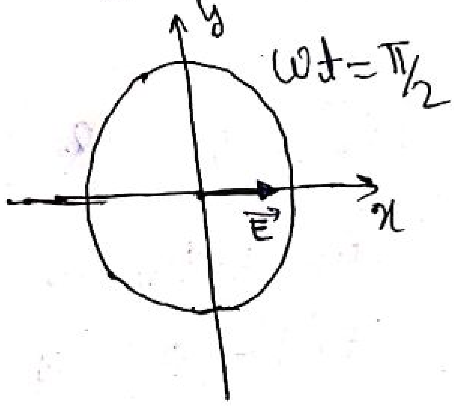
if $\omega t = 0$ $\quad |\vec{E_x}| = 0$ & $|\vec{E_y}| = -E_y$

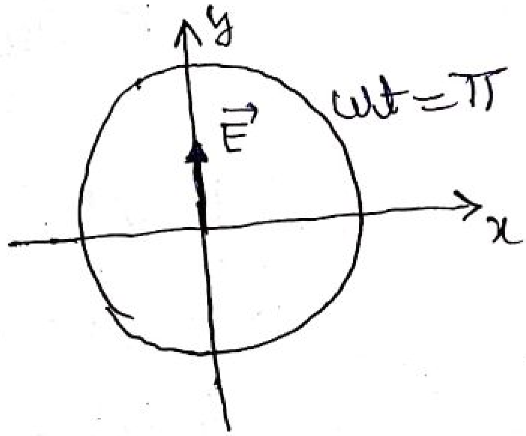
$\omega t = \pi/2$ $\quad |\vec{E_x}| = E_x$ & $|\vec{E_y}| = 0$

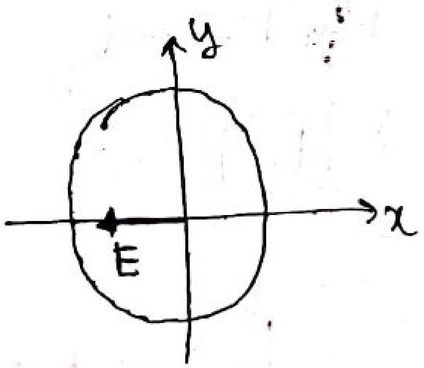
$\omega t = \pi$ $\quad |\vec{E_x}| = 0$ & $|\vec{E_y}| = E_y$

$\omega t = \frac{3\pi}{2}$ $\quad |\vec{E_x}| = -E_x$ & $|\vec{E_y}| = 0$









It is clear that, from these fig. that when two component of E_x and E_y of equal amplitude have a constant non-zero phase difference of $\pi/2$, the wave is said to be <u>Right circularly polarized</u>

Electromagnetic waves in a Conducting Medium

let, us Consider the propogation of e.m. waves in a Conducting medium. According to Ohm's law the current density in a Conductor is proportional to the e.f.

$$\vec{J}_f = \sigma \vec{E} \quad \text{——— (1)}$$

maxwell's equation -

(i) $\vec{\nabla} \cdot \vec{E} = \rho_f / \epsilon$

(ii) $\vec{\nabla} \cdot \vec{B} = 0$

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ ——— (2)

(iv) $\vec{\nabla} \times \vec{B} = \mu \sigma \vec{E} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}$

from eq^n of Continuity -

$$\vec{\nabla} \cdot \vec{J}_f = -\frac{\partial \rho_f}{\partial t} \quad \text{——— (3)}$$

Using (i) & (3), we can write

$$\frac{\partial \rho_f}{\partial t} = -\sigma \cdot (\vec{\nabla} \cdot \vec{E})$$

$$\frac{\partial \rho_f}{\partial t} = -\frac{\sigma}{\epsilon} \rho_f \quad \underline{\text{or,}} \quad \frac{\partial \rho_f}{\partial t} + \left(\frac{\sigma}{\epsilon}\right) \rho_f = 0$$

$$\boxed{\rho_f(t) = e^{-(\sigma/\epsilon) t} \, \rho_f(0)} \quad \text{——— (4)}$$

This eq^n (4) indicates that any free charge density $\rho_f(0)$ dissipates in a time '$\tau' = \sigma/\epsilon$.'

for, a <u>perfect Conductor,</u>

'$\sigma = \infty$' & $\tau = 0$

and, for a poor Conductor

$$\tau \leq 1/\omega$$

$$\boxed{\tau \geq 1/\omega}$$

When, $\rho = 0$ maxwell's eq^n can be written as

(i) $\vec{\nabla}\cdot\vec{E} = 0$

(ii) $\vec{\nabla}\cdot\vec{B} = 0$

(iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ ——— (5)

(iv) $\vec{\nabla}\times\vec{B} = \mu\epsilon\frac{\partial\vec{E}}{\partial t} + \mu\sigma\vec{E}$

Let, us apply curl to eq^n (iii) & (iv) of eq^n (5)

we get,

$$\nabla^2\vec{E} = \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} + \mu\sigma\frac{\partial\vec{E}}{\partial t}$$

$$\& \ \nabla^2\vec{B} = \mu\epsilon\frac{\partial^2\vec{B}}{\partial t^2} + \mu\sigma\frac{\partial\vec{B}}{\partial t} \quad \text{——— (6)}$$

The solutions of these eq^{ns} are

$$\vec{E}(z,t) = \tilde{E}_0 \cdot e^{i(\tilde{k}z-\omega t)}$$

$$\& \ \vec{B}(z,t) = \tilde{B}_0 \cdot e^{i(\tilde{k}z-\omega t)} \quad \text{——— (7)}$$

for the present case, wave no. K is a complex quantity. let,

$$\tilde{k} = k + i\kappa$$

and, $\tilde{k}^2 = \mu\epsilon\omega^2 + i\mu\sigma\omega$

Where

$$k = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}+1\right]^{1/2}$$

$$\kappa = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}-1\right]^{1/2} \quad \text{——— (8)}$$

the real part of $\tilde{k}$ (wave no.) determines the propogation speed, wave length and refractive index:

$$v = \frac{\omega}{k}, \ \lambda = \frac{2\pi}{k}, \ n = \frac{ck}{\omega}$$

V.imp — Imaginary part of w.no. results in attenuation of wave. the distance it takes to reduce the amplitude by a factor of $1/e$ is called the 'Skin depth' (d)

(decreasing amplitude with increasing distance)

it represents by $\boxed{d = \frac{1}{k}}$

$\%e$

Skin depth is a measure of How far e.m. wave penetrates into the Conductor.

the attenuated plane waves satisfies eqⁿ ⑥, and $\vec{E}$, polarized along x-direction are:

$$\vec{E}(z,t) = \tilde{E}_0 e^{-kz} e^{i(\tilde{k}z-\omega t)} \hat{x} \quad \text{—— (9)}$$

$$\text{and, } \vec{B}(z,t) = \tilde{B}_0 e^{-kz} \cdot e^{i(\tilde{k}z-\omega t)} \hat{y}$$

$\tilde{K}$ Can be expressed as $\tilde{K} = K \cdot e^{i\phi}$ —— (10)

Where $|K| = |\tilde{K}| = \sqrt{k^2 + k^2}$

$\tan\theta =$

$$= \omega\sqrt{\mu\epsilon\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}}$$

$$\& \quad \phi = \tan^{-1}\left(\frac{k}{K}\right) \quad \text{—— (11)}$$

According to eqⁿ (9) the Complex amplitudes of $\vec{E}$ & $\vec{B}$ are related by,

$$B_0 \cdot e^{i\delta_B} = \frac{K \cdot e^{i\phi}}{\omega} \cdot E_0 \cdot e^{i\delta_E}$$

this means, that the e.f. & m.f. are not in phase i.e. $\delta_B - \delta_E = \phi$ —— (12)

the m.f. lag behind the e.f. The

The real amplitude of e.f. & m.f. $\vec{E}$ & $\vec{B}$ are related by,

$$\frac{B_0}{E_0} = \frac{K}{\omega} = \sqrt{\mu\epsilon\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}}$$

finally, the e.f. & m.f of an e.m. wave inside a Conducting medium can be expressed as,

$$\boxed{\begin{aligned} \vec{E}(z,t) &= \tilde{E}_0 \cdot e^{-kz} \cdot \cancel{e^{i(\tilde{k}z-\omega t+\delta t)}} \cos(\tilde{k}z - \omega t + \delta_E)\ \hat{x} \\ \vec{B}(z,t) &= \tilde{B}_0 \cdot e^{-kz} \cdot \cos(\tilde{k}z - \omega t + \delta_E + \phi) .\ \hat{y} \end{aligned}}$$

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